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Determination of Electric Field in Non Homogeneous Dielectric Media with Cylindrical Geometry using the Thermal Step Method

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Abstract: This work is concerned with a method for determining the electric field across coaxial insulating structures with variable permittivity submitted to dc voltage. An analytical expression of the capacitive current issued by such a structure when subjected to a thermal stimulus is derived, and the calculation of the field repartition from the acquired signal is presented. Applications of the method to engineering components are brought into focus.

INTRODUCTION

It is well known that electric charge accumulates at dielectric interfaces, leading to electric field distortion able to induce overstress and to promote breakdown. Because combinations of dielectrics with different properties are frequently encountered in electrical and electronic applications, there has been an increasing interest over the last decade for determining electric charge and field distribution in non homogeneous dielectric structures.

Starting from the late 90's, various attempts have been made to use pressure methods (the pulsed electro acoustic method (PEA) in particular) for analyzing multi-dielectrics, laminates, or samples with variable permittivity at large [1-5]. During several years, little attention has been paid to the fact that the PEA method does not directly provide the space charge distribution within the sample if the latter is composed of different insulation layers. Recent works have shown that the extraction of charge and field repartitions in such cases is a delicate task, as taking into account the acoustic properties of the different layers and the occurrence of multiple reflections of the acoustic waves at interfaces require complex modelling, calibration and/or simulation [4-5]. Thermal techniques [6-8] are also applicable in such situations. In the case of the thermal step method (TSM) used on a flat sample with variable permittivity and thermal parameters, it has been shown that the analytical expression of the measured signal is not very different from the one corresponding to a homogeneous sample [9]. This allows considering the use of procedures for determining the electric field similar to those used for homogeneous samples.

The present paper deals with cylindrical insulating structures with variable permittivity. An analytical expression of the capacitive current issued from a non homogeneous coaxial structure crossed by a thermal wave while submitted to a dc voltage is first deduced. A procedure for determining the electric field distribution is then proposed and analyzed, and applications are discussed.

THEORY

Let us consider a coaxial structure of length L , of internal radius r_i and of external radius r_e (Fig. 1). The inner and outer surfaces of the cylindrical crown ($E1$ and $E2$, of radii r_i and r_e) are considered as conducting (electrodes), while the media placed between r_e and r_i is insulating. The permittivity ϵ of the insulating material of thickness $d = r_e - r_i$ varies with the radius r : $\epsilon = \epsilon(r)$. Due to the radial symmetry of the sample ($L \gg r_e$), the electric field is assumed as radial: $E = \vec{E}(r)$.

We assume that the sample contains a charge of value Q_0 , considered as constant during the experiment and uniformly distributed across an infinitely small circular layer of thickness dr and of coordinate r (Fig. 1). The voltage drop across the sample is noted U , and the sample is placed at a temperature T_0 . Because the system is in electrostatic equilibrium, under the principle of electrostatic influence the charge Q_i situated in the bulk of the insulator at the abscissa r induces at electrodes charges, noted Q_{i1} and Q_{i2} , such as:

$$Q_i + Q_{i1} + Q_{i2} = 0 \quad (1)$$

Let us now consider a cylindrical surface GCI of radius $u \in (r_i, r_e)$ and of length L (Fig. 1). By applying Gauss' theorem on GCI , we get

$$E(u) = (Q_{i1} + Q_i) / (2\pi u \cdot \epsilon(u)) \quad (2)$$

The same theorem applied to a cylinder similar to GCI but of radius $u \in (r_e, r)$ gives

$$E(u) = Q_{i2} / (2\pi u \cdot \epsilon(u)) \quad (3)$$

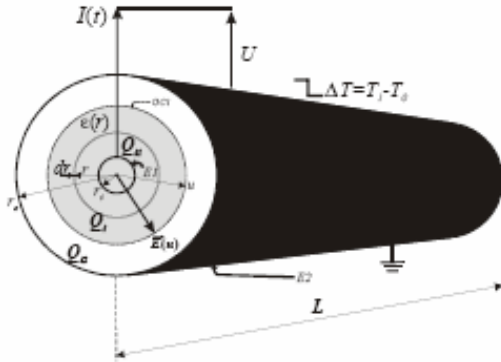


Figure 1. Diagram of a coaxial insulating structure with inhomogeneous permittivity and containing space charge, to which a thermal step is applied. E_1 , E_2 : electrodes, GC1: Gauss' cylinder used in Eq. (2).

The electric field circulation across the sample is

$$\int_{\gamma} E(u) \cdot du = U \quad (4)$$

From (1) to (4), we get

$$Q_n = -Q \left(\int_{\gamma} \frac{du}{ue(u)} + 2\pi IU \right) / \int_{\gamma} \frac{du}{ue(u)} \quad (5)$$

When a temperature step $\Delta T(u = r_0, t = 0) = T_1 - T_0$ is applied on the outer face of the specimen, the diffusion of the thermal front $\Delta T(u, t) = T(u, t) - T_0$ across the sample gives rise to local variations of the permittivity $\epsilon(u)$ and of the elementary radius du (expansion or contraction). The new permittivity and elementary displacement can be written (in first order) as $\epsilon(u)[1 + \alpha_\epsilon(u) \cdot \Delta T(u, t)]$ and $du[1 + \alpha_r(u) \cdot \Delta T(u, t)]$, where $\alpha_\epsilon(u) = (1/\epsilon) \times (\partial\epsilon/\partial\Delta T(u, t))$ is the ratio in variation of the material permittivity with the temperature and $\alpha_r(u) = (1/u) \times (\partial u/\partial\Delta T(u, t))$ its thermal expansion coefficient. Consequently, the quantity $du/(ue(u))$ becomes

$$\frac{du}{ue(u)} \rightarrow \frac{1 + [\alpha_\epsilon(u) - \alpha_r(u)] \Delta T(u, t) - \alpha_\epsilon(u) \alpha_r(u) \Delta T^2(u, t)}{1 - \alpha_\epsilon^2(u) \Delta T^2(u, t)} \quad (6)$$

Because in practice α_ϵ and α_r are less than 10^{-4} and $\Delta T < 30$ K, we have $\alpha_\epsilon(u) \alpha_r(u) \Delta T^2(u, t) \ll 1$ and $\alpha_\epsilon^2(u) \Delta T^2(u, t) \ll 1$. With these approximations and by putting $\alpha(u) = \alpha_\epsilon(u) - \alpha_r(u)$, one finally gets:

$$\frac{du}{ue(u)} \rightarrow \frac{du}{ue(u)} [1 + \alpha(u) \cdot \Delta T(u, t)] \quad (7)$$

Using this observation, equation (5) becomes:

$$Q_n = -Q \frac{\left[\int_{\gamma} \frac{du}{ue(u)} + \int_{\gamma} \frac{\alpha(u) \cdot \Delta T(u, t)}{ue(u)} du \right] \left[\int_{\gamma} \frac{du}{ue(u)} - \int_{\gamma} \frac{\alpha(u) \cdot \Delta T(u, t)}{ue(u)} du \right]}{\left[\int_{\gamma} \frac{du}{ue(u)} \right]^2 - \left[\int_{\gamma} \frac{\alpha^2(u) \cdot \Delta T(u, t)}{ue(u)} du \right]} + \frac{2\pi IU \left[\int_{\gamma} \frac{[1 - \alpha(u) \cdot \Delta T(u, t)] du}{ue(u)} \right]}{\left[\int_{\gamma} \frac{du}{ue(u)} \right]^2 - \left[\int_{\gamma} \frac{\alpha^2(u) \cdot \Delta T(u, t)}{ue(u)} du \right]} \quad (8)$$

The practical values of the α coefficient (quoted above) allow writing the following inequalities:

$$\left[\int_{\gamma} \frac{\alpha(u) \cdot \Delta T(u, t)}{ue(u)} du \right] \left[\int_{\gamma} \frac{\alpha(u) \cdot \Delta T(u, t)}{ue(u)} du \right] \quad (9)$$

$$\leq \alpha_{\max}^2 \cdot \Delta T_0^2 \cdot \int_{\gamma} \frac{du}{ue(u)} \cdot \int_{\gamma} \frac{du}{ue(u)} \ll \int_{\gamma} \frac{du}{ue(u)} \cdot \int_{\gamma} \frac{du}{ue(u)}$$

$$\left[\int_{\gamma} \frac{\alpha^2(u) \cdot \Delta T(u, t)}{ue(u)} du \right]^2 \leq \alpha_{\max}^2 \Delta T_0^2 \left[\int_{\gamma} \frac{du}{ue(u)} \right]^2 \ll \left[\int_{\gamma} \frac{du}{ue(u)} \right]^2$$

Using these inequalities, the expression of Q_n becomes

$$Q_n = -Q \left[\frac{\int_{\gamma} \frac{du}{ue(u)} + \frac{1}{\int_{\gamma} \frac{du}{ue(u)}} \int_{\gamma} \frac{\alpha(u) \Delta T(u, t)}{ue(u)} du}{\int_{\gamma} \frac{du}{ue(u)} - \int_{\gamma} \frac{\alpha(u) \Delta T(u, t)}{ue(u)} du} + \frac{2\pi IU \int_{\gamma} \frac{[1 - \alpha(u) \Delta T(u, t)] du}{ue(u)}}{\left[\int_{\gamma} \frac{du}{ue(u)} \right]^2} \right] \quad (10)$$

The current appearing in the external circuit when the temperature diffuses across the sample is due to the variation in the influence charges, i.e. $I(x, t) = -\partial Q_n / \partial t = \partial Q_c / \partial t$. Thus, in the case of a single charge Q_c , we have

$$I(x, t) = Q_c \left[\frac{1}{\int_{\gamma} \frac{du}{ue(u)}} \int_{\gamma} \frac{\alpha(u)}{ue(u)} \frac{\partial \Delta T(u, t)}{\partial t} du}{\int_{\gamma} \frac{du}{ue(u)} - \int_{\gamma} \frac{\alpha(u) \Delta T(u, t)}{ue(u)} du} + \frac{2\pi IU \int_{\gamma} \frac{\alpha(u)}{ue(u)} \frac{\partial \Delta T(u, t)}{\partial t} du}{\left[\int_{\gamma} \frac{du}{ue(u)} \right]^2} \right] \quad (11)$$

Let us put:

$$f(r,t) = \frac{1}{\int_{\eta}^{\xi} \frac{du}{ue(u)}} \int_{\eta}^{\xi} \frac{\alpha(u)}{ue(u)} \frac{\partial \Delta T(u,t)}{\partial t} du - \left[\frac{du}{ue(u)} \right]_{\eta}^{\xi} \int_{\eta}^{\xi} \frac{\alpha(u)}{ue(u)} \frac{\partial \Delta T(u,t)}{\partial t} du$$

$$r(r) = \left[\int_{\eta}^{\xi} \frac{du}{ue(u)} \right]_{\eta}^{\xi} \int_{\eta}^{\xi} \frac{\alpha(u)}{ue(u)} \frac{\partial \Delta T(u,t)}{\partial t} du \quad (12)$$

$I_i(r, t)$ can be written as:

$$I_i(r, t) = Q_i \cdot f(r, t) + 2\pi IU \cdot P(t) \quad (13)$$

In the general case of a space charge distribution of density $\rho(r)$, the external current $I(t)$ is due to the contribution of the entire charge $Q_i = \rho(r)dv = \rho(r)2\pi r dr$ contained by the sample ($dv = 2\pi r dr$ is the volume element). Consequently:

$$I(t) = \int_V Q_i \cdot f(r, t) + 2\pi IU \cdot P(t) = 2\pi I \int_{\eta}^{\xi} r \rho(r) f(r, t) dr + 2\pi IUP(t) \quad (14)$$

where V is the specimen volume.

By using the Poisson equation in cylindrical coordinates $\rho(r) = (1/r) d[re(r)E(r)]/dr$, the current expression becomes:

$$I(t) = 2\pi I \int_{\eta}^{\xi} \frac{d}{dr} [re(r)E(r)] f(r, t) dr + 2\pi IUP(t) \quad (15)$$

Integration by parts gives:

$$I(t) = 2\pi I \left[[re(r)E(r)f(r, t)]_{\eta}^{\xi} - \int_{\eta}^{\xi} re(r)E(r) \frac{\partial f(r, t)}{\partial r} dr \right] + 2\pi IUP(t) \quad (16)$$

Because $f(r, \eta) = f(r, \xi) = 0, \forall t$, the current expression is reduced to :

$$I(t) = -2\pi I \int_{\eta}^{\xi} re(r)E(r) \frac{\partial f(r, t)}{\partial r} dr + 2\pi IUP(t) \quad (17)$$

For calculating $\partial f(r, t)/\partial r$ we use the following integral formula:

$$\frac{\partial}{\partial r} \left[\int_{\eta}^{\xi} h(u, t) \cdot du \right] = \frac{\partial}{\partial r} [H(\xi, t) - H(\eta, t)] = -\frac{\partial H(r, t)}{\partial r} = -h(r, t) \quad (18)$$

leading to:

$$\frac{\partial f(r, t)}{\partial r} = -\frac{1}{\int_{\eta}^{\xi} \frac{du}{ue(u)}} \frac{\alpha(r)}{re(r)} \frac{\partial \Delta T(r, t)}{\partial t}$$

$$+ \frac{1}{re(r)} \left[\int_{\eta}^{\xi} \frac{du}{ue(u)} \right]_{\eta}^{\xi} \frac{\alpha(u)}{ue(u)} \frac{\partial \Delta T(u, t)}{\partial t} du$$

$$re(r) \frac{\partial f(r, t)}{\partial r} = -\frac{\alpha(r)}{\int_{\eta}^{\xi} \frac{du}{ue(u)}} \frac{\partial \Delta T(r, t)}{\partial t} + P(t) \quad (19)$$

Thus, the relation (17) becomes:

$$I(t) = \frac{2\pi I}{\int_{\eta}^{\xi} \frac{du}{ue(u)}} \int_{\eta}^{\xi} \alpha(r)E(r) \frac{\partial \Delta T(r, t)}{\partial t} dr - 2\pi I \left[\int_{\eta}^{\xi} E(r) dr \right] P(t) + 2\pi IUP(t) \quad (20)$$

With the change in variable $u = r, du = dr$ and because $\int_{\eta}^{\xi} E(r) dr = U$, one gets:

$$I(t) = \frac{2\pi I}{\int_{\eta}^{\xi} \frac{dr}{re(r)}} \int_{\eta}^{\xi} \alpha(r)E(r) \frac{\partial \Delta T(r, t)}{\partial t} dr \quad (21)$$

The quantity before the integral sign being the capacitance of the sample, we get the following analytical expression:

$$I(t) = C \int_{\eta}^{\xi} \alpha(r)E(r) \frac{\partial \Delta T(r, t)}{\partial t} dr \quad (22)$$

or

$$I(t) = -C \int_{\xi}^{\eta} \alpha(r)E(r) \frac{\partial \Delta T(r, t)}{\partial t} dr \quad (23)$$

The above expressions are obviously valid both under dc voltage and in short-circuit, as short-circuit is the particular case when $U = 0$.

COMPUTATION OF THE ELECTRIC FIELD

If we compare equation (22) to the expression established in the case of a homogeneous cylindrical insulation, i.e. [10]:

$$I(t) = \alpha C \int_{\eta}^{\xi} E(r) \frac{\partial \Delta T(r, t)}{\partial t} dr, \quad (24)$$

we note only one difference: in the case of the non homogeneous structure, the α factor moves under the integral sign. Consequently, the computation of the product $\alpha(r)E(r)$ in a non homogeneous structure may be achieved in the same manner as the field distribution $E(r)$ in a homogeneous sample. After determining the product $\alpha(r)E(r)$, the electric field distribution $E(r)$ across the inhomogeneous sample can be calculated by dividing this product by $\alpha(r)$. The space charge repartition $\rho(r)$ can then be computed with the Poisson equation.

To illustrate this procedure, in the absence of experimental data, the case of a polyethylene-insulated medium voltage cable for which the permittivity is constant across two thirds of the insulation and varies steeply across one third of the dielectric was treated by simulation. We assumed for the cable insulation the following dimensions: external radius $r_e = 14.55$ mm, internal radius $r_i = 10.55$ mm, and length $l = 19$ cm. This corresponds to dimensions frequently encountered in insulations used for 240 mm² ac medium voltage cables. The calculations have been made by considering that the polarization-type electric field distribution from Fig. 2 has been established across the cable insulation.

The temperature variation across the cable due to the arrival of a cooling liquid at -5°C on the external semicon of the cable initially at 25°C , corresponding to the experimental set up of the TSM presented in [11], has been calculated and used. The attenuation and the delay of the thermal step due to the characteristics of the experimental bench and to the cable external semicon layer have been taken into account as described in [11].

The law of variation illustrated in Fig. 3 was considered for the α parameter. It has been inspired by laws likely to be characteristic to water-treed cable insulation [12]. By using the above quoted parameters, the thermal step current from Fig. 4 has been obtained. The current was processed using the series technique described in [11] to obtain the product $\alpha(r)E(r)$, which has then been divided by the $\alpha(r)$ data from Fig. 3. This resulted in the field distribution from Fig. 5.

A good agreement between the assumed and the computed field curves (Fig. 2 and 5) can be noticed. The differences are essentially due to the fact that, for illustrating the possibilities of the processing technique, only five series terms have been used in the computation; the resolution can be increased with more terms [11].

The presented results show the applicability of the calculation procedure to coaxial non homogeneous structures, provided that the thermal and dielectric properties along the r -axis are known.

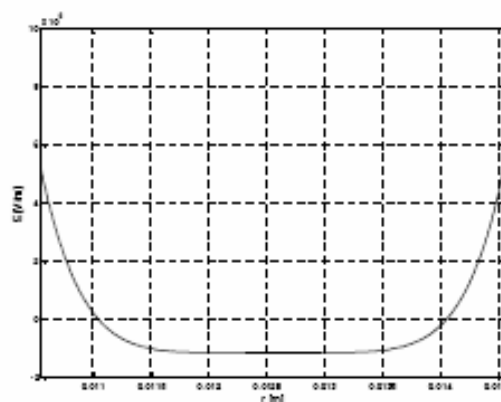


Figure 2. Assumed radial electric field distribution across the cable insulation.

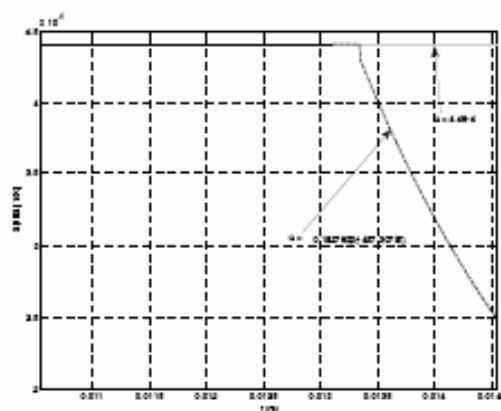


Figure 3. Assumed variation of the α parameter across the insulation.

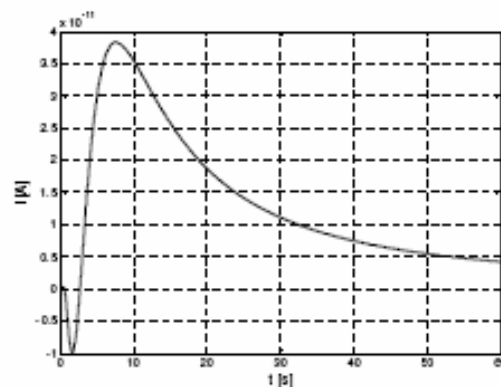


Figure 4. Thermal step current corresponding to the insulation with the radial field distribution from Fig. 2.

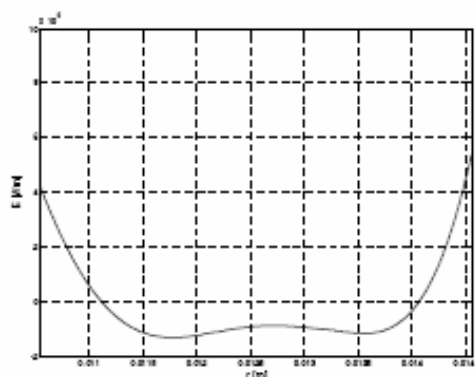


Figure 5. Electric field distribution computed from the thermal step current shown in Fig. 4.

FIELDS OF APPLICATION

A domain of choice for the proposed technique is high voltage dc power transport, where joints and terminations (made of coaxial layers of different materials [13]) are one of the leading concerns for cable manufacturers. In fact, space charge accumulation being a significant phenomenon in dc, the joint is a key part in power system reliability, because its geometry and structure makes it highly affected by space charge phenomena, which can initiate breakdown. Thus, the electric field distribution across the joints and its variation with time are important matters for design and lifetime estimation.

Another frequently encountered problem in power systems is the water treeing phenomenon appearing in medium voltage cables [14]. The development of these micro channels growing in the insulation under the effect of the water and of the electric field changes locally the field distribution by modifying the local permittivity and by introducing ions from water. It has been shown that the result may be a significant local intensification of the electric field, favouring electrical tree initiation and premature breakdown [12, 15]. A better knowledge of the real electric field values across the insulation would help understanding and modelling water tree growth in medium voltage cable insulation, leading to the set up of reliable non destructive methods for assessing the insulation operational state.

Because multi-dielectrics cylindrical insulations are widely encountered in electrical engineering, various other applications are possible. They may include insulating structures for high voltage modules used in medical applications, power electronics, space and airborne modules etc.

CONCLUSION

An analytical expression of the capacitive signal issued by a non homogeneous coaxial structure when submitted to a temperature variation in the r -direction has been deduced. It has been shown that the obtained expression is close to the formula corresponding to the case of a homogeneous sample, thus enabling the use of similar electric field computing methods, provided that the variation of thermal and electrical parameters along the r -axis is known. This allows considering the application of the technique to multi-dielectrics axisymmetric structures like cable joints and water-treeed cables.

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